On maximal connected I-spaces

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Abstract

Recall that a topological space is called *maximal connected* if it is connected but it has no connected strict expansion (a finer topology on the same set). A topological space is called an *I-space* if the set of all non-isolated points is closed discrete, equivalently if the space is a union of an open discrete subset and a closed discrete subset.

We will investigate maximal connected spaces in the realm of I-spaces, i.e. *maximal connected I-spaces*. Several properties of topological spaces are equivalent to being an I-space given the space is maximal connected. These include being scattered, having dense subset of isolated points, or being a union of finitely many discrete subsets. Also, a characterization of maximal connected I-spaces would generalize the known characterization of finitely generated maximal connected spaces.

We consider several constructions preserving maximal connectedness – so-called *I-compatible tree sums* and *ultrafilter I-extensions* – and we show how to build a subclass of maximal connected I-spaces by iteratively applying these constructions. We call these maximal connected I-spaces *inductive*. Finally, we give an example of a non-inductive maximal connected I-space.